Math 55 Discussion problems 20 Apr

1. Is (S, R) a poset if S is the set of all people in the world and $(a, b) \in R$, where a and b are people, if
   (a) a is no shorter than b?
   (b) a weighs more than b?
   (c) $a = b$ or a is a descendant of b?
   (d) a and b do not have a common friend?
2. Draw the Hasse diagram for inclusion on the set $P(S)$, where $S = \{a, b, c, d\}$.
3. Answer these questions for the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$.
   (a) Find the maximal elements.
   (b) Find the minimal elements.
   (c) Is there a greatest element?
   (d) Is there a least element?
   (e) Find all upper bounds of $\{2, 9\}$.
   (f) Find the least upper bound of $\{2, 9\}$, if it exists.
   (g) Find all lower bounds of $\{60, 72\}$.
   (h) Find the greatest lower bound of $\{60, 72\}$, if it exists.
4. Give a poset that has
   (a) a minimal element but no maximal element.
   (b) a maximal element but no minimal element.
   (c) neither a maximal nor a minimal element.
5. (a) Show that the least upper bound of a set in a poset is unique if it exists.
   (b) Show that the greatest lower bound of a set in a poset is unique if it exists.